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IVAN GRIGOR'YEVICH BUBNOV
On the 125th anniversary of his birth
(6(18) January 1872–13 March 1919)†

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The eminent Russian scientist Ivan Grigor'yevich Bubnov is widely known for his remarkable research in mechanics and mathematics. He is also known for his fundamental research on shipbuilding and the design of submarine and surface vessels, and, in modern parlance, can be considered to have been the Chief Designer and Engineer of the Russian Navy.

Bubnov was born in Nizhnii Novgorod. After leaving secondary school, he enrolled in the shipbuilding section of the Kronstadt Technical Institute of the Naval Department, from which he graduated with distinction in 1891. Until 1894 he was a junior shipbuilding officer's mate of the military port at St Petersburg, helping to construct the battleships *Sysoi Veliki* and *Poltava*. Between 1894 and 1896 he studied at the shipbuilding division of the Naval Academy, where he subsequently taught right up until his death.

In 1898 he was appointed senior shipbuilding officer's mate of the Baltic Shipbuilding and Mechanical Yard, and he worked there until 1915. In 1903 he was designated head of the shipbuilding drawing-office of the Naval Technical Committee, and served as head of the Model Basin between 1908 and 1915.

Bubnov loved teaching and devoted much time and effort to it and the Naval Academy (1896–1919) and St Petersburg Polytechnic Institute (1903–1915). He lectured on a wide variety of problems in mathematics, mechanics and shipbuilding, but his main efforts were directed to writing a course on the theory of elasticity and structural mechanics of ships and the design of ships. His published courses [1,

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2] were motivated by practical ship design and construction and include new ideas on mechanical models of rods, plates and shells, and on methods of estimating the strength of ships. They reflect his previous published work on pitching in waves, the difficulty of launching a ship, making ships unsinkable (the design of bulkheads), weight analysis of a vessel, and standards of acceptable stress in shipbuilding.

After defending his dissertation in 1904, Bubnov was awarded the degree of Advanced Student in Applied Mechanics. In 1906 he was made extraordinary professor, in 1910 ordinary professor, and in 1916, Emeritus Professor of the Naval Academy. From 1912 he was Major General of the Corps of Ship Engineers.

He died in the prime of his creative output.

Design and development. The construction of an effective model on which to estimate the stresses and strains and the strength of submarine and surface ships exposed to global and local effects is a difficult scientific and technical problem. Bubnov worked tirelessly on this problem and produced 48 ship designs.

Between 1900 and 1908 he designed the first Russian naval submarines—the *Del'fin*, *Kasatka*, *Minoga*, *Akula*, *Bars* and *Morzh*. From the very beginning he was able to produce an advanced design—a prototype which was capable of systematic modification. Bubnov's boats had the best tactical and technical specifications of the time. The *Kasatka* participated in military action during the Russian–Japanese war, and the *Akula* and *Bars* took part in World Wars I and II.

The contribution that Bubnov made to the construction of new Russian surface vessels after the Russian–Japanese war is inestimable. He won first place in the International Contest in Battleship Design, organized by the Russian Admiralty in 1907, after which he made calculations and published a project. Two years later, work was begun under his supervision on the famous battleships *Sevastopol* (subsequently, after the October Revolution, the *Paris Commune*), *Petropavlovsk* (*Marat*), *Poltava* (*Oktyabr'skaya Revolyutisya*), *Gangut* and, some time later, another three battleships. Battleships like the *Sevastopol* incorporated in their design a longitudinal (stringer) framing, which soon acquired the name of "the Russian System". This was a major step in the revision of the ships' architecture inherited from the time of wooden vessels, and made it possible to increase the longitudinal strength of the ship considerably and reduce the mass of the cross-framing and consequently use a stronger bottom plating to reinforce the ship. In addition, Bubnov proved that it was possible to divide a ship into sections, indicated the best rig stowage arrangement and the best choice of leakproof bulkheads and decks. He made estimates of the strength of a ship's frame and established standards for acceptable stresses in design components. Later (in 1911) he produced designs for the heavy cruiser *Izmail*. His calculations on battleships were published in five volumes [3]. At the time, these surface vessels were widely thought to be the best in the world.

It is worth mentioning Bubnov's paper [4] on general methods of ship design, which includes a discussion of an original prototype based on his idea that "any design which is suitable for implementation is but a development, modification or improvement of an existing type".

Research on the mechanics of a deformable body. Bubnov's article entitled "On the stresses in a ship's bottom plating due to water pressure" [5] has played an enormous part in the development of various areas of the mechanics of deformable media.

This was the first study ever of finite deflections of a long thin elastic rectangular plate under uniform transverse pressure and lateral uniform compression. He investigated the resistance of a sheet with small deflections, for which the Germain–Lagrange theory applies, with finite deflections, when bending and extension (compression) of the sheet must be taken into account, and with large deflections, when the sheet operates as a membrane. This was the first time that the last two cases had been examined.

Further on in the paper he derives the equations of finite deflection of a thin axisymmetric elastic circular plate under uniform pressure assuming equal radial and annular forces in the plane of the plate; the differential equation of the problem turned out to be linear and was integrated. This result was subsequently used as a basis for the theory of non-linear plates of general shape and generated an extensive literature. Bubnov himself studied the case of supported and clamped edges, analysed the case of a membrane and extended it to the elastoplastic behaviour of a material. After studying the behaviour of a plastic plate at a supported contour, he showed how this clamping device becomes a hinge. Thus he devised the concept of a plastic hinge long before Kazinskii (1914), who is generally acknowledged as its inventor. We note also that Bubnov was the first to introduce the concept of a secant modulus in plasticity theory [5]. He solved the problem of the deflection of rigid plates with rigidly clamped edges, representing the deflection by a double trigonometric series. It is also worth noting that Bubnov obtained the equation for a circular plate [5] before Föppl's work on a membrane (1907) and

Kármán's for flexible plates (1910). Thus Bubnov's case is a consequence of the Föppl–Kármán non-linear equations, but was studied earlier.

Finally in the same paper [5] Bubnov studied the behaviour of a long thin shallow elastic cylindrical panel of infinite size under uniform transverse pressure and uniform lateral compression, derived the equations of the problem and showed that the deflection (curvature) can change abruptly under compressive lateral forces. Not only is this the first solution ever of the non-linear problem for a shell and an exact solution at that, but it also describes a new mode of shell stability loss involving an abrupt change (snapping), not Euler instability at low deflections, but stability loss at finite deflections. This process was later developed for slightly curved rods by Timoshenko (in 1925 for a heated bimetal and in 1935 for a uniform rod). Note that Kármán and Tsien studied the snapping of a spherical shell under uniform radial pressure and of a cylindrical shell under uniform longitudinal compression in 1939 and 1941 respectively.

In Part II of his course Structural Ship Mechanics [2] Bubnov derives a method of calculating the strength of plates reinforced by closely situated stiffeners. This is now known as the method of constructive anisotropy.

Problems of the stability and transverse bending of loaded plates will be considered below in a discussion of Bubnov's orthogonalization method.

We now come to Bubnov's paper [6], in which he analyses the well-known solution of Euler loss of stability of a thin elastic circular cylindrical shell under uniform hydrostatic pressure. His assumptions that small terms can be neglected and that stability loss if accompanied by the formation of a large number of waves in the radial direction were widely used during the 1930s in the linear and non-linear theory of shells (Mushtari, Donnell, Marguerre *et al*).

Bubnov's method of solving differential equations. In 1911 Timoshenko presented a monograph [7] in a competition for the Zhuravskii prize, named after the famous engineer. This had been set up in 1902 but had still not been awarded. Four of the written reviews of this work were published [8] and judging took place (for the first and last time) in May 1911. Bubnov's review ([8], pp. 33–36) contains the orthogonalization method in its first formulation, not later than May 1911.

Bubnov believed ([8], pp. 33–36) that problems of the stability of rods and plates should not be solved by energy methods, as Timoshenko had proposed [7], but by introducing into the left-hand side of the differential equation $L(x,y) = 0$ an approximate expression for the deflection which satisfies the contour conditions and is a convergent series, $w(x,y) = a_1\varphi_1(x,y) + a_2\varphi_2(x,y) + \dots$ ($\varphi_k(x,y)$ are functions of the coordinates and a_k are constants), multiplying the resulting expression $L^*(x,y)$ by $\varphi_k(x,y)dxdy$ and integrating over the area of the middle surface (a, b are the dimensions of the plate in plan), where

$$L^*(x,y) \neq 0, \quad \int_0^a\int_0^b L^*(x,y)\varphi_k(x,y)dxdy = 0 \quad (k = 1,2,\dots)$$

These orthogonalization conditions for the expression $L^*(x,y)$ and the approximated function $\varphi_k(x, y)$ can be used to obtain solutions of problems for a selected number of terms of the series for $w(x,y)$. The critical load is determined by equating the determinant of the coefficients of the unknowns a_1, a_1, ... to zero. He used this method to calculate the "force of a column" for Euler's problem from the differential equation of Euler stability and the critical force of a uniformly unilaterally compressed thin elastic rectangular plate with hinged edges from Navier's differential equation, using a double trigonometric series.

In Section 22 of Part II of the course Structural Ship Mechanics [12, pp. 515–544; reprinted; 9, pp. 190–218] Bubnov applies the orthogonalization method (using a double Navier series) to a number of problems on the effect of uniform transverse pressure and a linearly distributed load parallel to one of the sides in the plane of a rectangular plate, and also considers the effect of uniform transverse pressure and uniform shear forces on the contour of a rectangular plate with freely supported edges using the Saint Venant equations. In both cases (for these new and difficult problems of stability) the critical forces are determined as a function of the ratio of the sides, and a detailed analysis of the computation of the critical forces is presented. In the first case Bubnov reduces the partial differential equation to an ordinary differential equation, which he solves by the orthogonalization method, and in the second he uses the orthogonalization method to reduce the partial differential equation immediately to a system of linear algebraic equations.

Bubnov developed his method as an alternative to the energy method, and so did not discuss some of the difficulties that were later encountered in using it. For example, how does one solve the problem if not all the contour conditions for the approximated function are satisfied? What should one do if the order of the equation is artificially increased or reduced? In particular, Bubnov obtained an exact solution of the differential equation for an axisymmetric circular plate with finite deflections using Bessel functions. But had he used the orthogonalization method, the equation would have had to be orthogonalized with respect to the angle of inclination of the normal (the first derivative with respect to the deflection) rather than with respect to the deflection, since he began with one integration of the initial fourth-order equation of the problem.

While Bubnov's method was first met with suspicion, it has long been used very widely, and very effectively, in various mechanical and physical models, particularly from non-linear equations and systems. I present a detailed discussion of Bubnov's method in my monograph [11].

The creative legacy that Bubnov left (his work is reprinted in [9, 10] has made a huge contribution to science and technology and ensured that his name is remembered in the history of our country.

Bubnov designed and constructed the entire national submarine fleet of the time and his surface ships were original and used as a model for ship design elsewhere. The science that Bubnov created—structural ship mechanics, has also been used in the development of aircraft design. He was the first to develop the theory of the stability loss for elastic surfaces at finite deflections and to analyse other problems of the non-linear resistance of plates. He was the first to formulate the equations of finite deflections of thin surfaces (subsequently known as the Föppl–Kármán–Marguerre equations), and the first to introduce the concept of a secant modulus and develop the method of constructive anisotropy. The orthogonalization method of integrating differential equations alone would have been enough to ensure the inclusion of Bubnov's name among the bright stars of science.

Of his remarkable disciple, Academician A. N. Krylov wrote: "The services that Ivan Grigor'yevich Bubnov rendered make him unforgettable in the annals of our country, and his work will long continue to be used in the construction of both military and commercial vessels".

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